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IMPLEMENTATION OF THE FINITE ELEMENT METHOD IN CIVIL ENGINEERING IN BULGARIA

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Keywords: finite element method, implementation, structural mechanics, TUBA, prof. Karamanski

ABSTRACT

This paper describes the main steps in the introduction of the Finite Element Method (FEM) in Bulgaria, as well as its use in the educational process at the University of Architecture, Civil Engineering and Geodesy, Sofia. After a brief description of the introduction of the idea of the method in the mid-20th century in the world, its introduction and development in Bulgaria is described, along with the development of computing technology. Attention is paid to Prof. Todor Karamanski, his doctoral dissertation and activities related to the application of computational methods in civil engineering. The works of colleagues from the University dedicated to FEM are also briefly listed.

1. Introduction and basics of FEM development

Building structures are designed by creating appropriate mechanical and mathematical models. Depending on the dimensions of the structural components, rod, plane or volume models are used. The classical approach to studying planar and volumetric models is to use continuum mechanics – a branch of mechanics that deals with the deformation of and transmission of forces through materials modeled as a continuous medium. The theoretical background of continuum mechanics lies in the physics of continuous materials, and can be divided into two main branches:

- Solid mechanics;
- Fluid mechanics.

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Fluid mechanics is concerned with the mechanics of fluids (liquids, gases, and plasmas). Solid mechanics studies the behavior of solid materials, especially their motion and deformation under the action of forces, temperature changes, and other external impacts. Its practical applications include various fields of mechanical engineering, materials science, medical informatics, geophysics, and other areas of geosciences. It is most widely used in structural mechanics, for structural design purposes. Structural mechanics analysis has three approaches:

- energy methods;
- flexibility method;
- direct stiffness method.

FEM can be considered as a variant of a direct stiffness method, especially in the analysis of frame structures. In a solid environment, the approach is more general.

The procedure involves establishing relationships between the mean values of various quantities associated with infinitesimal elements. The equations governing the behavior of the entire domain are obtained by allowing the element sizes to approach zero.

The idea of FEM can be defined as the opposite of the classical approach. The procedure of FEM begins by examining the properties of finite-sized elements. The equations describing the continuum are employed to describe the properties of these elements, but the dimensions of the elements remain finite in the analysis.

Its development can be traced back to work by Alexander Hrennikoff [1], published in 1941. He divided the solution domain into a grid of lattice structure, which was the earliest form of mesh discretization. In 1943 Richard Courant [2] used the Rayleigh–Ritz method with a trial function defined on finite triangular sub-domains, which is a primitive form of the finite element.

In the civil engineering community, the article representing the beginning of finite elements is considered to be [3], (see Fig. 1). Its authors (M. J. Turner, R. W. Clough, H. C. Martin and L. J. Topp) employ both a local approximation (of the partial differential equations of linear elasticity) and the use of assembly strategies essential to finite element methodology.

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Stiffness and Deflection Analysis of Complex Structures

M. J. TURNER,* R. W. CLOUGH,† H. C. MARTIN,‡ AND L. J. TOPP**

Figure 1. The title page of the iconic paper [3]

In [4] Ray W. Clough infix the phrase “Finite Element Method”, which clearly reveals the essence of the method. Among the creators of the FEM is Johann Hadji Argyris – a pioneer of computer applications in science and engineering in the mid 1960s. Some of his famous articles are [5 and 6].

The well-known Rayleigh-Ritz method is a direct numerical method for eigenvalue approximation. It is named after Lord Rayleigh and Walter Ritz. The method consists of approximating an infinite-dimensional linear operator by finite-dimensional compression. It often exists as an option in civil engineering software. Its algorithm is usually called the Ritz-Galerkin method when used in the context of the FEM.

One of the early pioneers of the FEM is Olgierd Zienkiewicz. He recognized the potential of the method for solving problems outside the field of solid mechanics. He has published over 500 scientific articles and has served on the editorial boards of over 20 other scientific journals.

Funding from state and private organizations also influences the development of FEM. In the late 1960s, the United States government funded the aerospace industry to create the NASTRAN program. Originally developed for NASA, it did not have any functionality at the time that allowed for building graphical models or networks. All input and output data for the program were in the form of text files.

One adaptation of FEM is the finite element TUBA, which is specifically designed to handle plate problems and provide high-order approximations. The TUBA family (“Triangular Unit-Box Approach”) of plate finite elements is adopted for the discretization of the displacement vector field. They were initially proposed in the 1950s. These triangular elements, with necessarily straight sides in the reference configuration, provide continuous approximations, a sine qua non property for a consistent discretization of the Kirchhoff–Love model. The TUBA family of elements also includes elements without intermediate nodes, i.e., without nodes located at the edges. These elements are known for their ability to handle bending and membrane behavior, offering a robust approach to modeling structures like plates, shells, and shear walls. In general, triangular elements are more flexible for complex geometries. On the other hand, quadrilateral elements can be more computationally efficient in regular domains and offer higher accuracy for certain problems.

Numerous other scientists and engineers have contributed worldwide to the development of the FEM by different approaches.

2. Biographical data about Prof. Todor Karamanski

Todor Karamanski was born on 03 January 1932 in the town of Kyustendil. He joined the Department of Structural Mechanics in 1954. He has been a doctor (PhD) since 1965, an associate professor since 1970, and a professor since 1979. He was the Dean of the Faculty of Structural Engineering in the period 1991 – 1994. Other positions he was known for are:

- Long-time member of the Specialized Scientific Council on Structural Mechanics at the Bulgarian Higher Attestation Commission.
- Chairman of the Union of Builders and Designers in Bulgaria (UCB) 1996.
- Long-time chairman of the regional branch of the Chamber of Engineers in the Investment Design.
- Long-time lecturer in the disciplines taught at the Department of Structural Mechanics.
- Author of numerous textbooks and teaching aids.
- With a great contribution to the development of the disciplines FEM and Computer Analysis of Structures.

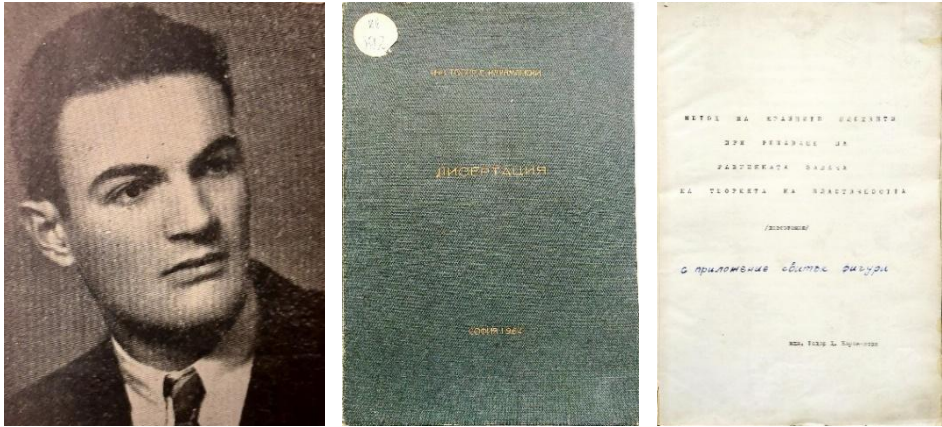


Figure 2. Todor Karamanski as graduate along with his PhD book [7]

As a proven specialist in the field of Structural Mechanics and the application of numerical methods for the study and design of structures, he has participated in the management of a number of structures in the field of education. Prof. T. Karamanski participated in the design and preparation of a number of expert assessments of buildings and facilities of national importance (Kozloduy Nuclear Power Plant, National Palace of Culture and many others).

3. First dissertation on the topic FEM in Bulgaria

In 1965, Prof. T. Karamanski defended his dissertation “Finite Element Method in Solving the Plane Problem of the Theory of Elasticity” [7] (see Fig. 2), where for the first time in our country the principles of numerical methods in the FEM style were applied.

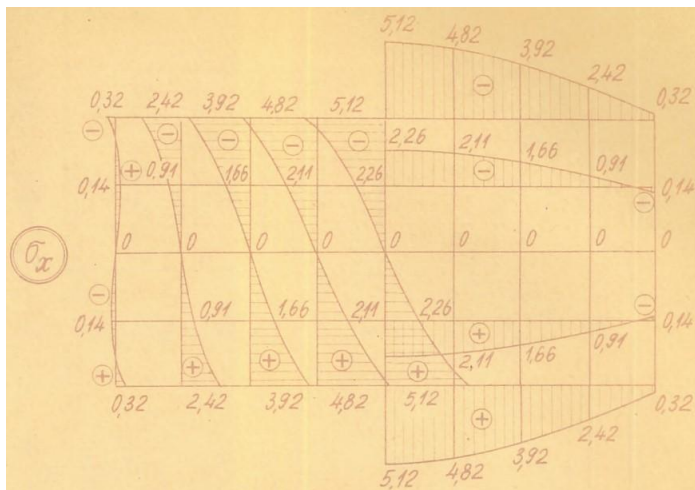


Figure 3. Image of figure 3.6 (normal stress by sections) from [7]

A little later, the Bulgarian Agricultural Academy was delivered an electronic digital machine “ODRA 1204” with an ALGOL compiler. Prof. T. Karamanski began programming on

it, and then conducted training courses for interested lecturers from the department and other departments, as well as for students. At that time, the first programs for static analysis of structures appeared. The program for constructing influence lines for internal forces in continuous beams with variable cross-section, developed by Prof. T. Karamanski, was used in the design of the Asparuhov Bridge (Varna city). It was implemented on the "ODRA 7100" of the Ministry of Education, where the input and output information carrier was punched tapes.

In 1979, a team of highly qualified specialists under the leadership of Prof. Todor Karamanski developed the "Stadyps" [8] program system, which found its initial application in the design of the National Palace of Culture. The first versions of [8] were written in Fortran. With the development of computer technology, the program was recompiled, and also passed through different operating systems: OS, DOS, Windows. Since 1980, the department has been conducting "Practical Training in Structural Statics", which covers various software that has been updated over the years.

In his dissertation, Eng. Todor Karamanski makes numerous comparisons of solutions to the same example using different methods (see Fig. 3). As part of the conclusions, the advantages of FEM are highlighted, especially in comparison with the finite difference method (FDM). The original image of the results table (see Fig. 4) shows the clarity of the study. Table 8.9 presents the most significant results from the solutions of several examples, under different loads.

Таблица 8.9

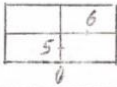
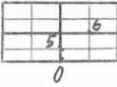

Гъстота на мрежата	Метод	При- мер	a_0		b_5		c_6		Необ- ходи- ми урав- нения
			Стой- ност	Греш- ка в %	Стой- ност	Греш- ка в %	Стой- ност	Греш- ка в %	
 2λ. 2h	метод на крайните разлики	8.22	3,2423	38,8%	0,4053	40,2%	0,8106	38,7%	0
	метод на крайните елементи	8.17	3,7358	29,5%	0,5000	26,3%	0,9339	29,3%	1
 4λ. 4h	метод на крайните разлики	8.25	4,5372	14,4%	0,6147	9,36%	1,0540	20,2%	2
	метод на крайните елементи	8.20	5,2602	0,77%	0,6956	2,57%	1,3118	0,69%	9
	с използване на уедрени клетки	8.21	5,2020	1,86%	0,7236	6,69%	1,2898	2,35%	3
 6λ. 3h	Метод на крайните разлики	8.27	5,0804	4,16%	0,6541	3,56%	1,2090	8,46%	9
Действителни стойности, получени по метода на тригонометричните редове			5,3006	-	0,6782	-	1,3208	-	-

Figure 4. Image of Table 8.9 from [7]

Among the calculation methods used is the Ritz method. The conclusion is made that due to the significant complexity (resource-consuming) of the solution using variational methods in general, these methods cannot be considered appropriate for solving the planar problem.

For example, to solve the example shown in the dissertation, it is necessary to perform 114 arithmetic operations (addition, subtraction, multiplication and division) by FEM, while to solve the system, which also has 9 equations, but is composed using the FDM, 320 arithmetic operations are required, i.e. nearly three times more.

The computational work in the FDM is also increased because, after solving the system of equations from which the values of the Airy function at the characteristic points are determined, additional computational work is required to determine the values of the stresses. In the FEM, the computational work ends with the solution of the system of equations.

When comparing FEM with other methods, the PhD work also considers an important characteristic of systems of algebraic equations, namely their conditioning. We say that a system is ill-conditioned if small changes in the coefficients and free terms lead to large changes in the values of the unknowns. Ill-conditioned systems need to be calculated with increased accuracy, since in the course of the solution there is a great loss of significant digits. One of the conclusions is that FDM leads to worse-conditioned systems of equations.

The advantages explored in this dissertation mark the beginning of the application of FEM in Bulgaria.

4. Global and national recognition of Bulgarian research in the field of FEM

After successfully defending his dissertation, Todor Karamanski continued his work in the same field, publishing his paper [9] in 1967. This article is still available today (see Fig. 5) and was cited in 1984 in [10], in a French scientific work, where the interaction between a reinforced concrete frame and hollow brick masonry under seismic stress is investigated. There [10] they appreciate his work, as the first to use the FEM in the analysis of filled frames (in original “cadres remplis” or C. R., see Fig. 6), by considering that the frame supports only axial forces – while the connection between the frame and the filling is never destroyed – that the interaction between the frame and the filling is carried out by means of shear stresses.



Tall Buildings
The Proceedings of a Symposium on Tall Buildings with Particular Reference to
Shear Wall Structures, Held in the Department of Civil Engineering, University of
Southampton, April 1966



1967, Pages 455-463

CALCULATING INFILLED FRAMES BY THE METHOD OF FINITE ELEMENTS

TODOR KARAMANSKI

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Publisher Summary

The basic method for solving problems of the mechanics of uninterrupted media

**Figure 5. Appearance of [9] on the internet nowadays,
along with its digital object identifier**

The unknowns are the constraints at the nodes. Approximations of the constraint functions between nodes are made using numerical quadrature formulas (2nd or 3rd degree parabolas).

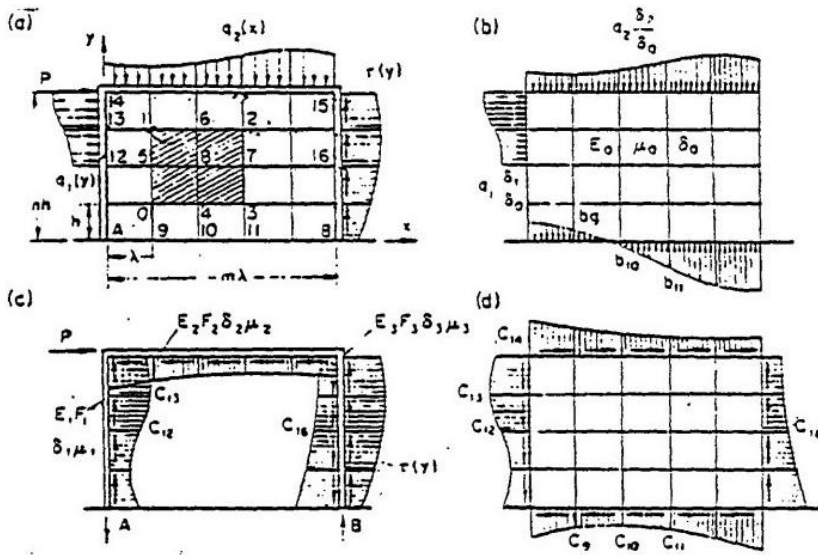


Fig. 1.8 - MODELE ANALYTHIQUE DE KARAMANSKI

Figure 6. Image of a cited figure in [10] by name

5. Activities of other members of the department staff

Prof. Karamanski has supervised 18 Bulgarian and foreign doctoral students who occupy a worthy place in the science and education of their countries.

Among the colleagues from Todor Karamanski's generation who developed the topic of FEM, we can highlight Banko P. Bankov and Yuliana V. Pavlova. They co-authored two books [11 and 12] on the method during their time as leading lecturers in "Statics of Structures" in UACEG.

Professor Bankov has specialized in "Limit state analysis of plane frames" and "Introduction to FEM" at the Industrial Technical Center for Metal Construction in Paris. Assoc. Prof. Pavlova – fundamentals of FEM at the Norwegian University of Science and Technology (NTNU) in Trondheim, Norway. The specializations were fruitful and resulted in the publishing of books in the field of FEM [11, 12], which at that time were very valuable for engineers and postgraduates. PhD students of Professor Bankov, who developed dissertations with the application of FEM, were Anita Handruleva and Lyuben Lyubenov, who later became lecturers at other universities.

In [6] the displacement functions for plate elements subjected to bending are given. Attention is paid to both triangular and quadrilateral elements that either satisfy all the conditions for kinematic compatibility or are insufficient with respect to the continuity of the gradient normal to the edge.

The original 1968 publication showing the TUBA3 element [6] contained some errors in the transformation associated with the normal derivative of one side [13]. These errors were identified and corrected by Assoc. Prof. Pavlova and Prof. Karamanski [14] in 1979. This paper is read and cited around the world even today, for example [15 and 16].

The book [17] is a textbook that includes the topics of FDM, FEM, and other numerical methods, such as the boundary element method.

The book [18] includes a chapter dedicated to the analysis of plane stress elasticity problems using FEM. The main stages of the implementation of displacement-based FEM are shown. Examples are given in rectangular and triangular finite (shell) elements for solving thin rigid plates.

Among the doctoral students, Alexander Traykov continues his career as a lecturer at UACEG, while simultaneously developing design activities, applying his skills in the field of the finite element method. He is currently a leading lecturer in the “Computer-Aided Analysis of Structures” discipline. Another lecturer who was a doctoral student of Prof. Karamanski is Konstantin Savkov Kazakov. He is the author of many textbooks and articles, for example [19 and 20].

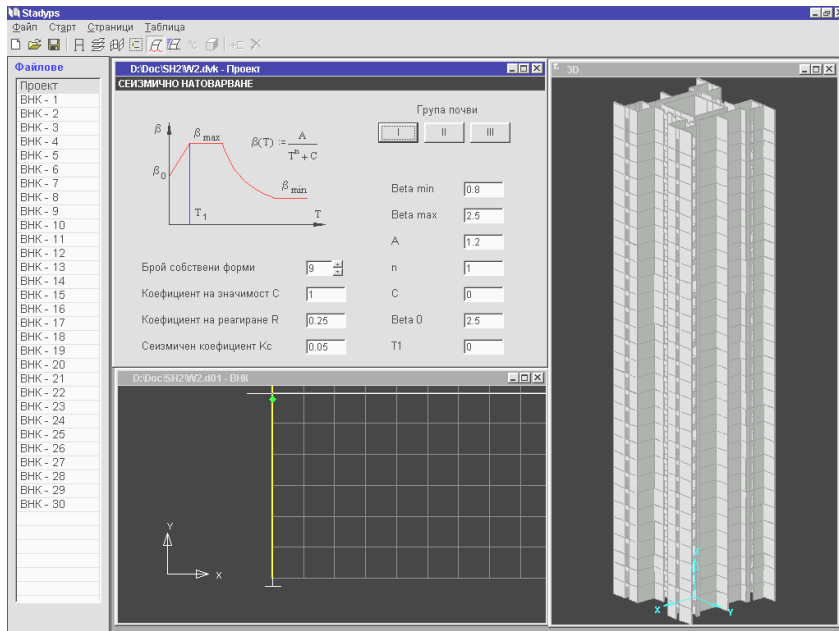


Figure 7. Screenshot of the software [21]

Other colleagues from the department also publish both research papers and coursebooks related to FEM.

In the period around the 1980s, many professors from the Department of Structural Mechanics developed software, running under the DOS operating system, for calculating structures based on FEM: Ivo Baychev, Todor Vasilev, Alexander Atanasov Popov, Ivan Markov. Over the next 2 decades, the program [8] ran under OS and DOS on various machines: PC, XT 286, PC/AT 386, etc. It was implemented in some of the largest architectural and engineering design organizations like Glavproekt, Chimcomplex Engineering, Metalproekt, Mostconsult etc. It was applied in the design of thousands of residential, public and industrial buildings. Some of the most important between them are the National Palace of Culture and the cantilever bridge on km +48,500 of Hemus highway. In 2000, a version [21] running under MS Windows was developed, see Fig. 7.

Currently, Eng. Ganchovski is working on a dissertation at the Department of Structural Mechanics with the help of his program [22]. In his paper [23] he reported that the results from his own plane frame analysis program, developed in [22], completely match those obtained from [8]. [22] is powered by a comprehensive math engine with built-in support for units, vectors,

matrix calculations and numerical methods. Users can develop their own computational procedures in a relatively simple custom programming language. Some of the already developed examples include linear and nonlinear static and dynamic analysis of plane trusses, frames, plane stress and plates. [22] was first published in 2021 as free and open-source software and currently is used by tens of thousands of engineers worldwide.

Ivan Markov has compiled and published a textbook on FEM, which has been used by UACEG students for years [24]. Ivan Markov together with Alexander Taushanov have conducted research and published articles describing the stiffness matrix of finite elements involving spring supports [25, 26].

Dobromir Dinev and Ivan Markov have demonstrated the application of FEM in the field of medicine by modeling the femur in the case of using a screw implant due to a fracture [27]. Dinev is a lecturer in the subject FEM (in English) and conducts research analysis of beams strengthened by composite strips [28]. Sonia Parvanova and Georgi Pekov Vasilev are also lecturers in the discipline of FEM. All listed colleagues can use a variety of modern structural analysis software, including in elective courses.

Nowadays, the capabilities of electronic computing equipment and software allow us to use models with many degrees of freedom, which makes entering input information easier, and the visualization of results can usually be customized.

6. Conclusions

The FEM is the most successful numerical method used to solve mechanics and engineering problems. In its beginnings, the FEM did not reach the practice of engineering on a massive scale due to the non-availability of computers in engineering studies and the requirement of in-depth knowledge not only of the technique and the relevant mathematical models but also of computational programming. Today, the situation is completely different, as modern personal computers easily support powerful, easy-to-use general-purpose programs.

It is used to evaluate the safety and integrity of structures such as bridges, buildings, and dams. Professor Todor Karamanski is a pioneer in the implementation of FEM both in the educational process at UACEG and in the engineering practice in Bulgaria.

Almost every doctoral dissertation in the field of structural mechanics involves the use of FEM. All lecturers who have defended their doctorates in the Department of Structural Mechanics have excellent knowledge of FEM, which they teach on to their civil engineering students.

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ВНЕДРЯВАНЕ НА МЕТОДА НА КРАЙНИТЕ ЕЛЕМЕНТИ В СТРОИТЕЛНОТО ИНЖЕНЕРСТВО В БЪЛГАРИЯ

А. Таушанов¹

Ключови думи: метод на крайните елементи, внедряване, строителна механика, метод на крайните разлики, триъгълен краен елемент, МКЕ

РЕЗИЮМЕ

Тази статия описва основните стъпки при въвеждането на метода на крайните елементи (МКЕ) в България, както и използването му в образователния процес в Университета по архитектура, строителство и геодезия, София. След кратко описание на налагането на идеята за метода в средата на 20-и век в света, е описано неговото въвеждане и развитие в България, заедно с развитието на изчислителната техника. Обръща се внимание на проф. Тодор Карамански, неговата докторска дисертация и дейности, свързани с приложението на изчислителните методи в строителното инженерство. Накратко са изброени и трудове на колеги от университета, посветени на МКЕ.

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