ГОДИШНИК НА УНИВЕРСИТЕТА ПО АРХИТЕКТУРА, СТРОИТЕЛСТВО И ГЕОДЕЗИЯ СОФИЯ

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ARE THE CLASSIC WEIGHTS IN PRECISE LIVELLING ALWAYS THE BEST CHOICE?

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Keywords: *adjustment, error propagation, precise levelling, weights*

ABSTRACT

Despite being in usage for more than 150 years the error accumulation in precise levelling has not yet been completely clarified. It is believed that the error accumulation in this method is proportional to the square root of the levelling distance. The first goal of this paper is to demonstrate that this belief is not always scientifically proved. The second aim is to show that it is likely that a better adjustment decision will be missed if inverse distance weighting with a power parameter equal to one is automatically applied. Using linear regression analysis, the measuring data of the Second Levelling of Finland is analyzed. An inadequacy of the relationship between the absolute values of the double run differences of the heights between the terminals of the levelling lines and their length is shown, which is due to a heteroscedasticity. In order to obtain a homoscedastic model other two regression models are constructed. Based on these results, the analyzed network is adjusted using three types of weights. The adjustment with traditional weights has produced significantly greater mean errors of the nodal benchmarks than both variants based on weights, which are functions of the sum of the absolute values of the section elevations in the lines.

1. Introduction

There are different approaches for determining the vertical position of a single point or a group of points in the geodetic practice. One of them is precise levelling. Due to its high accuracy

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this method has been used since the last decades of the 19th century for investigation of the recent land uplifts of the Earth's crust [1 – 3], among others. Other geodetic activities where the precise levelling takes place are setting of regional or continental vertical reference systems [4 – 6] estimation vertical datum offsets from Global Geopotential Models [7 – 9], intercontinental height datum relations [10], search of active geodynamic activities [11], etc. Thus, this method is essential for various scientific studies. The precise levelling is also applied widely in a vast area of engineering activities where the highest accuracy of vertical measurements is required [12, 13].

The development of the method in the last decades is focused on involving in practice digital levels, modern rods and rod comparators [14, 15], optimization of the measurement process by prior preparation [16], minimization of systematic effects [17, 18], etc.

Most of the best world practices were recommended in [19, 20] and were included in the new levelling instruction in Bulgaria [21].

Despite the above mentioned improvements, it seems that the precise levelling has reached the end of its development as a measuring method. For example, the mean error of the Third Levelling in Finland, calculated from the closing errors of the levelling loops, is $\pm 0{,}86$ mm/km^{0,5} [22]. The corresponding accuracy of the Second Levelling is $\pm 0{,}60$ mm/ km^{0,5} [23].

Identical results have been announced for Polish precise levelling campaigns [2].

Analyzing the mean errors obtained from the adjustment of the precise levelling networks of the participants of the United European Levelling Network (UELN) [5], one can see that only the Hungarian network has less mean error per unit weight than the Second Levelling of Finland, $\pm 0{,}47$ kgal.mm/km^{0,5} against $\pm 0{,}63$ kgal.mm/km^{0,5} [23], respectively. All of these networks were measured and adjusted decades after the Second Levelling of Finland. Therefore, if we want to develop precise levelling as a method we should reconsider some of the basic assumptions concerning errors and their accumulation in sections, lines and loops.

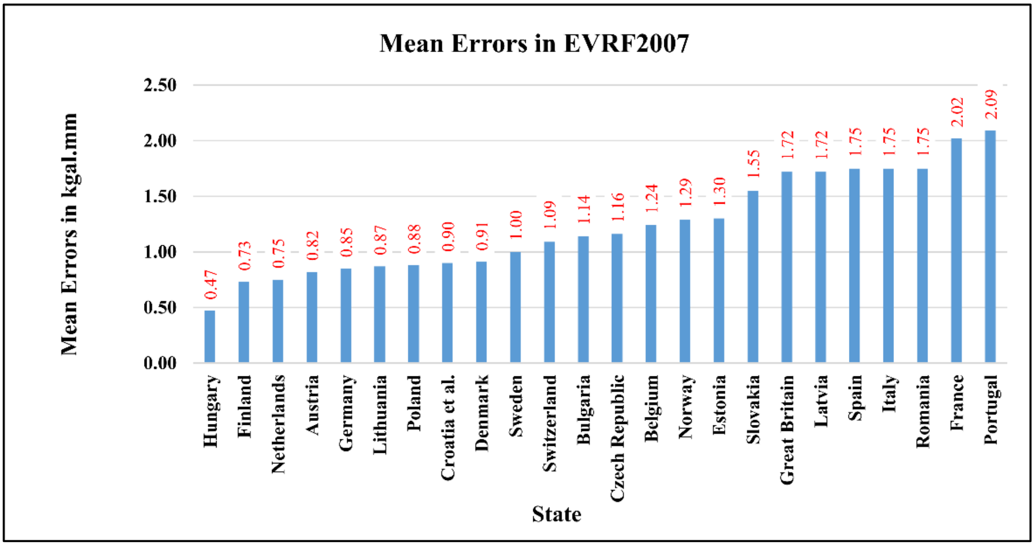

Figure 1. Ascending Ordered Mean Errors in EVRS2007 (Sacher *et al.* 2008, Table 2)

Looking at Figure 1, one can easily see that the terrain of the first nine states, except Austria, is mainly flat. The territories of all other states, which are pictured in Figure 1, are mountains and rougher terrain areas. Consequently, it is likely that there is a relationship between the accuracy of the networks and the relief of the territory, which they cover.

In fact, it has been shown [24] that the accumulation of the absolute values of the discrepancies in the Third Levelling of Bulgaria is more strongly related to the sum of the absolute values of the section elevations along the lines than to the length of the lines. This fact implies that the classic weights, which are recommended by [21], may not be the best choice concerning the relief of Bulgaria. This statement will be proved in a future paper.

It will be interesting to investigate whether or not the classic low accumulation of errors in precise levelling is met in a state with flat area. For this reason, the results of the Second Levelling of Finland will be analyzed. Additional reasons for choosing this network are the accessibility of the measured data [23, Table IX] and the extremely high accuracy of this network.

The questions we will discuss in this article are the following:

- How adequate is the proportional relationship between the discrepancies in the measured elevations and the lengths of the levelling lines in the Second Levelling of Finland?
- Are there any other weights that better fit the analyzed network?

2. Experimental and adjustment procedures

Models and weights

Let us have a levelling line, whose length is s km. If the mean error of the levelling per km is ε, the mean error of the difference in height between the terminals of this line is ε_s then, a relation among them can be given by equation (1) [1, p. 39].

$$\varepsilon_s = \pm\varepsilon\sqrt{s} \ . \tag{1}$$

Thus, the weight of the line in an adjustment is

$$P = p_0 \big/ s.\varepsilon^2 \ . \tag{2}$$

Let p_0 be a constant suitably selected. It is better to take ε as a constant in the entire network so $p_0 = \varepsilon^2$. Let $s = L$. Thus (2) turns in (3).

$$P = \frac{1}{L} \ . \tag{3}$$

The mean error ε_s can also be expressed by H_1 and H_2 which are the values of the elevation between the terminals of the line yielded in both measurements.

$$\varepsilon_s = \frac{H_1 - H_2}{2} = \pm\frac{D}{2} \ . \tag{4}$$

After putting (4) in (1), replacing $2\varepsilon = \text{const}$ and $s = L$, we derive the first model (5) which we are going to examine using regression analysis.

$$|D| = \text{const}\sqrt{L} \ . \tag{5}$$

Equations (3) and (5) represent the popular assumption of the most appropriate weights in an adjustment of levelling networks and the most correct low error accumulation in levelling, respectively. The relevance of (5) will be checked by regression analysis. Residuals derived from the analysis will be examined whether meet the basic assumptions of the ordinary least squared regression analysis. The relevance of (3) will be estimated by using t-tests paired two sample for means. The first sample will contain the mean errors of the nodal bench marks yielded by the use of the weights (3) and the other samples will be generated from the mean errors of the correspondent nodal bench marks but derived from adjustments which use nonconventional weights.

It is a well-known fact that levelling results obtained on routes with long and high inclinations are more affected by different kinds of systematic errors, e.g. refraction, inclination of the rods during the measurements, errors in the rod meter due to calibration or temperature changes [25, 26], etc. Levelling along inclined routes also increases the number of set ups. As a result, the effect of vertical movements of the tripod and rods is getting bigger. Also, the influence of the errors due to vertical movements of the common station rod is amplified. Consequently, the accumulation of double run measurements in the precise levelling should be better explained as a function of measured elevations. It seems reasonable the accumulation of errors to be referred with an accumulation of the absolute values of measured elevations in levelling sections $|h|$. So, our second model is (6).

$$|D| = \text{const} \sum |h| . \tag{6}$$

Based on model (6) and using the above logic we can construct (7).

$$P = \frac{1}{\left(\sum |h|\right)^2} . \tag{7}$$

In addition, we will also examine weights (8).

$$P = \frac{1}{\sum |h|} . \tag{8}$$

In order to increase the statistical power of our regression analysis all lines included in [23, Table IX] will be used.

Adjustment procedure

Our aim is to compare the results derived from an adjustment of the network of the Second Levelling of Finland with the weights given by equation (3) and both nontraditional weights (7) and (8). The configuration of the network can be found in [23, p. 11]. There are some slight differences between this study and the original one, namely:

- Loops IV and V are not included in the adjustment because of the fact that the entire information for lines 13, 14 and 17 was not published in (Kääriäinen 1966, Table IX).

- In order to obtain directly the weight coefficients Q_{ii} of the nodal bench marks the parametric adjustments are used here. The condition taken on (Kääriäinen 1953, p. 54) and the treatment of the weight coefficients described on (Kääriäinen 1953, p. 55 – 56) is also included in our calculations.

– The connections to the tide gauges are not included in our adjustment. For this reason, as a fundamental bench mark is chosen the bench mark in Qulunkylä instead of the Fundamental Bench Mark in Helsinki.

– Because of the simplicity, some lines between nodal bench marks are combined. For example, lines 31, 32 and 33, which take part in loop *X*, are joined in a common line. Lines 35, 36 and 37 are treated in the same manner. And so on. As a result, our network contains 26 nodal bench marks, where the nodal bench mark in Qulunkylä has a known height and other bench marks have unknown heights. Finally, the result network is consisted of 41 levelling lines.

In order to clarify utterly the adjustment procedure, a brief explanation is given below. Let v_{ij} are the corrections in the measured heights h_{ij} between bench marks *i* and *j* and their initial heights are H_i and H_j, respectively. Let x_i and x_j are the corrections of H_i and H_j. Then, our correction equations can be written as (9).

$$V_{ij} = \left(H_i + x_i\right) - \left(H_j + x_j\right) - h_{ij} = x_i - x_j + \left(H_i - H_j - h_{ij}\right) = x_i - x_j + f_{ij} \ . \quad (9)$$

Thus, in matrix form (9) can be presented by (10).

$$V = A.X + f \ . \quad (10)$$

In equation (10) *A* is an information matrix, which contains the coefficients forward the unknowns *x*, *X* is a vector of the unknowns *x* and *f* is a vector of the free members in (9). Using the above symbols, we can write our additional condition (11) and in matrix form (12).

$$\sum x_i = 0 \, . \quad (11)$$

$$B.X = 0 \, . \quad (12)$$

In (12) *B* is a size 25 vector of ones. Now our aim is to obtain the unknown *x* in accordance with conditions (12) and (13).

$$V^T PV \to \min \, . \quad (13)$$

In (13) *P* is a matrix of the weights. All members of *P* matrix are zeros except the members in the main diagonal, which are values of the weights of each levelling line calculated by (3), (7) and (8). To obtain the corrections v_{ij} and x_i we use equation (14), where *K* is a vector of correlates. In our adjustment we have only one correlate.

$$Q = V^T PV + 2K^T BX \ . \quad (14)$$

After substitution into Lagrange's equation $dQ / dV = 0$ and some matrix manipulations we yield (15).

$$A^T PAX + B^T K + A^T Pf = 0 \, . \quad (15)$$

Let $N = A^T PA$ be a normal matrix and $A^T Pf = F$. Thus, we yield our extended normal system (16) or (17) and after its solution we will obtain the unknown corrections *x*.

$$\begin{vmatrix} N & B^T \\ B & 0 \end{vmatrix} . \begin{vmatrix} X \\ K \end{vmatrix} + \begin{vmatrix} F \\ 0 \end{vmatrix} = 0 . \tag{16}$$

$$N_e . X_e + F_e = 0 . \tag{17}$$

Using equation (17) one can yield the corrections v_{ij}. The mean error of the weight unit *m* can be calculated by (18), where $r = n - k = 55 - 32 = 23$.

$$m^2 = \frac{[PVV]}{r} . \tag{18}$$

The matrix $Q = N_e^{-1}$ is an extended covariance matrix. The first $k = 32$ values in the main diagonal of *Q* are the inverse values of the nodal bench mark weights. These values are treated as is described in [1, p. 55 – 56] in order to receive the final values Q'_{ij}. Finally, the mean errors of the nodal benchmarks can be computed by (19).

$$m_i = m \sqrt{Q'_{ij}} . \tag{19}$$

3. Results

Regression analysis results

The results derived from the regression analysis of the models defined by equations (5) and (6) are given in Table 1.

Table 1. Regression analysis results

Results	**$\|D\| = \text{const } L^{0,5}$**	**$\|D\| = \text{const} \sum\|h\|$**
1	**2**	**3**
R	0,764	0,790
R^2	0,584	0,625
Adjusted R^2	0,577	0,618
Standard Error	5,137 mm	4,876 mm
Observations	146	146
F	203,269	241,451
Significance	$2,57.10^{-29}$	$1,36.10^{-32}$
Constant	0,9594 $\text{mm}/\sqrt{\text{km}}$	0,0333 mm/m

The residual plots of the investigated models (5) and (7) are pictured in Figure 1 and Figure 2, respectively.

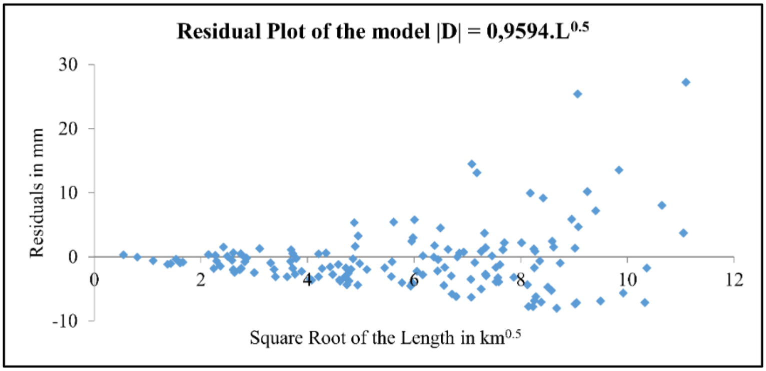

Figure 2. Residual Plot of the model |*D*| = const $L^{0,5}$

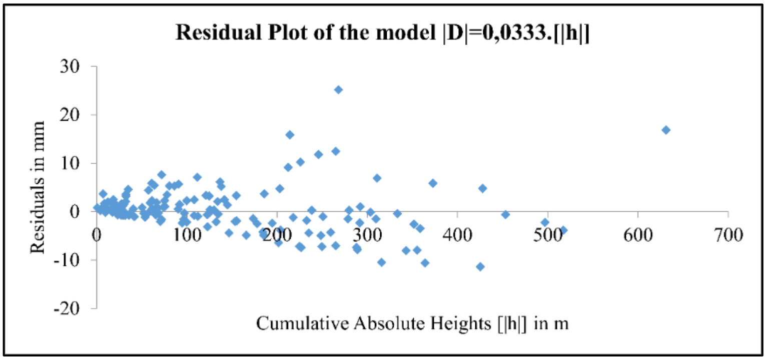

Figure 3. Residual Plot of the model |*D*| = const ∑|*h*|

Adjustment results

Figure 4 shows the mean errors of the nodal bench marks yielded from the adjustments of the Second Levelling of Finland by using weights (3), (7) and (8).

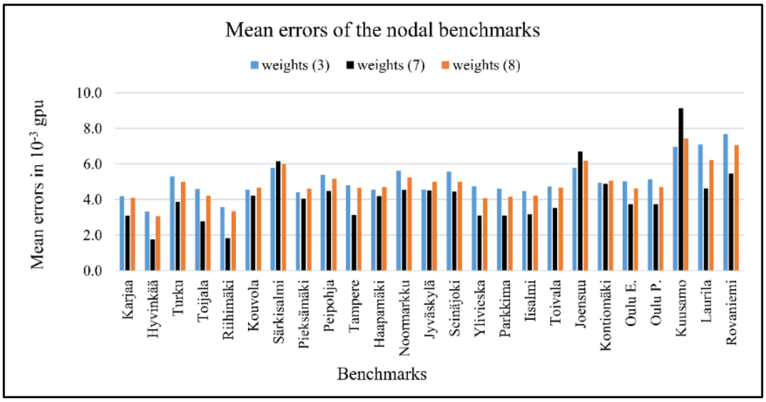

Figure 4. Mean errors of the nodal bench marks derived from the adjustments with weights (3), weights (7) and weights (8)

Hypothesis testing results

Table 2. t-Test: Paired Two Sample for Means using the samples of the mean errors of the bench mark derived from the adjustments with weights (3) and (7)

Description	$P = 1 / L$	$P = 1 / (\sum\|h\|)^2$
1	2	3
Mean	$5{,}101.10^{-3}$ gpu	$4{,}174.10^{-3}$ gpu
Median	$4{,}810.10^{-3}$ gpu	$4{,}060.10^{-3}$ gpu
Variance	$1{,}024.\ 10^{-3}$ gpu^2	$2{,}403.\ 10^{-3}$ gpu^2
Observations	25	25
Pearson Correlation	0,752	
df	24	
t Stat	4,488	
P(T<=t) one-tail	$7{,}63.10^{-5}$	
t Critical one-tail	1,711	
P(T<=t) two-tail	0,0002	
t Critical two-tail	2,064	

Table 3. t-Test: Paired Two Sample for Means using the samples of the mean errors of the bench mark derived from the adjustments with weights (3) and (8)

Description	$P = 1 / L$	$P = 1 / \sum\|h\|$
1	2	3
Mean	$5{,}101.10^{-3}$ gpu	$4{,}934.10^{-3}$ gpu
Median	$4{,}810.10^{-3}$ gpu	$4{,}700.10^{-3}$ gpu
Variance	$1{,}024.\ 10^{-3}$ gpu^2	$1{,}037.\ 10^{-3}$ gpu^2
Observations	25	25
Pearson Correlation	0,936	
df	24	
t Stat	2,301	
P(T<=t) one-tail	0,015	
t Critical one-tail	1,711	
P(T<=t) two-tail	0,030	
t Critical two-tail	2,064	

Table 4. t-Test: Paired Two Sample for Means using the samples of the mean errors of the bench mark derived from the adjustments with weights (7) and (8)

Description	$P = 1 / (\sum\|h\|)^2$	$P = 1 / \sum\|h\|$
1	**2**	**3**
Mean	$4{,}174.10^{-3}$ gpu	$4{,}934.10^{-3}$ gpu
Median	$4{,}060.10^{-3}$ gpu	$4{,}700.10^{-3}$ gpu
Variance	$2{,}403.\ 10^{-3}$ gpu^2	$1{,}037.\ 10^{-3}$ gpu^2
Observations	25	25
Pearson Correlation	0,916	
df	24	
t Stat	-5,13	
P(T<=t) one-tail	$1{,}49.10^{-5}$	
t Critical one-tail	1,711	
P(T<=t) two-tail	$2{,}99.10^{-5}$	
t Critical two-tail	2,064	

4. Discussion

One of the goals of this paper was to demonstrate the inadequacy of the classic assumption concerning the accumulation of discrepancies in precise levelling and consequently to raise a question about the choice of the most suitable weights used in adjustment of precise levelling networks.

Analyzing the residuals derived from the regression analysis of the model defined by equation (5), one can see a presence of heteroscedasticity. The residuals pictured in Figure 2 are fan-shaped, which is evidence of unequal variance of the residuals. This fact shows inadequacy of the classic model [27, p. 347, Fig. 11.3] in the case of the Second Levelling in Finland. The inadequacy of this model is also shown for the Polish Fourth Campaign [28, Fig. 7], as well as for the Bulgarian Third Levelling [24, Fig.1 and Fig.4]. The residuals pictured in [8, Fig. 6] also reveal a presence of heteroscedasticity when differences in elevations are tried to be described by the square root of a length.

Looking at Figure 3 it can be seen that the residuals of models (6) are more randomly distributed, and the presence of heteroscedasticity is more difficult to detect. Although Figure 3 reveals some signs of heteroscedasticity, the plotted residual in this figure is more closely to the patterns illustrated by Figure 11.2 than to Figure 11.3 in [27, p. 347], where the expected pattern on regression residuals is illustrated. Moreover, according to Table 1, the model based on equation (6) explains 62,5 % of the variance of the residuals while the classic model explains 58,4 % of the variance. Also, the standard error of model (6) is less than the standard error of the classic variant, 4,876 against 5,137 units.

Of course, when a plot of residuals of a regression model pictures inadequacy, it is not the best decision to trust the model standard error, F-statistic, etc. In order to avoid this trap, we will simply compare the mean errors of the nodal bench marks which were produced from adjustments using different weights.

Figure 4 clearly shows that using the nonconventional weights in adjustment of the network lead to less mean errors of the nodal bench marks than the classic weights given by equation (3).

Because of the fact that the mean error of a unit weight in our adjustments are dimensionally different and for this reason incomparable, the samples of the mean errors derived in the separate adjustments are compared by t-Test: Paired Samples for Means. The results are given in Table 2, Table 3 and Table 4. According to these tables the average of the mean errors yielded from the adjustment with the weights given by equation (3) are significantly greater than the averages of the mean errors obtained from the adjustments with both weights (7) and (8). Both tests are significant at 95 % confidence level. According to Table 5, there is also significant differences between the averages of the mean errors of the nodal bench marks yielded by weights (7) and (8).

It is important to be remarked that the median of the mean errors obtained by weights (7) is 4,06 mm and there is no mean error which exceeds 10 mm. Analyzing Table 2, one can count that only 4 out of 25 nodal bench marks have mean errors greater than 5 mm in the case of the adjustment with weights (7).

5. Conclusion

Are the classic weights in precise levelling always the best choice? The answer is 'no' since:

- An inadequacy of the relationship between the absolute differences $|D|$ produced from both measurements of the heights between the terminals of the lines on the one hand and the square root of the length of lines L on the other hand is detected in the most accurate levelling network in the world. Such inadequacy was also illustrated in [24], where the data of the Third Levelling of Bulgaria were analyzed. This inadequacy could explain the differences in the levelling accuracy calculated on the basis of discrepancies in sections, lines and loops.
- The models used to explain the accumulation of the absolute differences $|D|$ with the sum of the absolute value of the section elevations in lines $\sum|h|$ lead to more homoscedastic results.
- Nonconventional weight models (7) and (8) lead to approximately 20 % smaller mean errors of the nodal bench marks in the network of the Second Levelling of Finland than the most popular weights (3). This difference in the accuracy is significant at 95 % confidence level. The last fact implies that it is always possible to find a better weighing model than the classic one.
- It is likely that independent analyses of the highest order levelling networks of other states will confirm the above reveals.

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ВИНАГИ ЛИ КЛАСИЧЕСКИТЕ ТЕЖЕСТИ ЗА ИЗРАВНЕНИЕ НА ПЪРВОКЛАСНИ НИВЕЛАЧНИ МРЕЖИ СА НАЙ-ДОБРИЯТ ИЗБОР?

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Ключови думи: *прецизна нивелация, изравнение, предаване на грешки*

РЕЗЮМЕ

Независимо от широкото използване на прецизната нивелация повече от 150 години, законите за натрупване на грешки в измерените превишения все още не са напълно изяснени. Счита се, че точността на нивелачните измервания е правопропорционална на квадратен корен на пронивелираното разстояние. Една от целите на този доклад е да покаже, че това схващане невинаги е вярно. Втората цел е да се покаже, че е възможно по-добро решение да бъде пропуснато в случай на автоматично приемане на класическите тежести. При използване на регресионен анализ е установено, че натрупването на грешки във Втората нивелация на Финландия по-добре се обяснява с натрупването на измерените превишения в нивелачните линии, отколкото с корен квадратен от тяхната дължина. Предвид от този факт е извършено изравнение на анализираната мрежа с използването на три различни вида тежести. За анализараната мрежа е установено, че вариантът с използването на класическите тежести в нивелацията води до статистически значимо по-големи средни квадратни грешки на възловите репери в сравнение с двата варианта на изравнение, базирани на тежести, обратно пропорционални на сумата на абсолютните стойности на секционните превишения.

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