ГОДИШНИК НА УНИВЕРСИТЕТА ПО АРХИТЕКТУРА, СТРОИТЕЛСТВО И ГЕОДЕЗИЯ
СОФИЯ

Том Volume 56 | 2023 | Брой Issue 2

ANNUAL OF THE UNIVERSITY OF ARCHITECTURE, CIVIL ENGINEERING AND GEODESY
SOFIA

Получена: 14.12.2022 г.
Приета: 13.01.2023 г.

REDUCING ERROR PROPAGATION IN AN ADJUSTMENT OF PRECISE LEVELLING NETWORKS

A. Angelov[1], V. Cvetkov[2]

Keywords: *adjustment, error propagation, precise levelling*

ABSTRACT

The objective of this paper is to compare two modes of an adjustment of the Third Levelling of Bulgaria data. The first one is a parametric adjustment of the network as free. The second mode is a parametric adjustment with the condition that the sum of the corrections of the adjusted heights of the nodal bench in the network is equal to zero. Both variants use as a datum level the fundamental bench mark in Varna. The comparison of the results derived from both approaches shows that the mean errors in the adjusted heights of the nodal bench marks in the network obtained in the second variant are almost twice less than the corresponding errors produced by the first mode. It is also shown that the first variant produces a systematic increase of the mean errors of the adjusted heights with distancing from the datum point in Varna. Such systematic behavior was not detected in the second variant of the adjustment. The absolute differences in the adjusted heights of the bench marks also increase with distancing and exceed 10 mm for one half of the bench marks, which is higher than the obtained mean errors in the heights of the benchmarks in the second variant.

1. Introduction

Since February 2021 there is a new instruction [1] for creating, supporting and maintenance of precise levelling networks concerning the territory of Bulgaria. This document includes a dozen valuable prescriptions according to the measurement process and some

[1] Antonio Angelov, Assoc. Prof. Dr. Eng., Dept. “Applied Geodesy”, UACEG, 1 H. Smirnenski blvd., Sofia 1046, e-mail: angelov_fgs@uacg.bg

[2] Vasil Cvetkov, Assist. Prof. Dr. Eng., Dept. “Technology and Construction Management”, VSU “Lyuben Karavelov”, 175 Suhodolska St., Sofia 1373, e-mail: tzvetkov_vasil@abv.bg

systematic error corrections given in [2, 3], which had not taken place in the old instruction [4] but have been proven and widely used in the world, e.g. a refraction correction.

There is also some development in the part of the adjustment of precise levelling networks. The new instruction allows adjustment not only of normal height differences but also of geopotential differences, which was done in Finland a few decades ago [5].

Furthermore, a possibility of the usage of weights inverse to the number of stations along levelling lines is suggested, which was recommended in [6] obviously without any analysis of available levelling data [7]. There are investigations [8, 9] based on [7], which show that there is not any significant difference between the results produced in an adjustment with the usage of both (1) and (2) types of weights referring to the Bulgarian conditions.

$$w = \frac{1}{L}, \tag{1}$$

$$w = \frac{1}{N}. \tag{2}$$

In equations (1) and (2) L and N are the length of a levelling line and the number of stations in the line, respectively. This fact can be explained with a strong correlation between L and N. What is more, [1] decreases the maximum length of the distance between the level and rods from 40 m to 30 m. As a result, the correlation between L and N will increase, which implies that the usage of (2) will not be a huge leap ahead in terms of improving the adjustment methodology.

Fortunately, there is a decision from the middle of the 20th century, which produces almost twice less mean errors of nodal bench marks and helps to reduce the propagation of errors with distancing from a datum. Luckily, [1] does not specify what kind of the least squares methods to use for an adjustment of the highest order levelling networks, which gives opportunities to extend our experience [10] with the methodology explained in [11, p. 55]. The best decision should be applied in the adjustment of the Fourth Levelling of Bulgaria.

In order to illustrate the relevance of the Finnish approach, the data of the Third Levelling of Bulgaria will be used. Moreover, the original decision, which is a conditional least squares method with an additional condition, will be replaced with a parametric method with a condition, owing to some computational advantages, i.e. directly obtaining the accuracy of nodal bench marks in the network.

2. Adjustment procedures

2.1. General notes

There are a few publications related to the adjustment of the Third Levelling of Bulgaria. Probably the most popular and cited is [10], where briefly and frugally variants of the adjustment of the network are mentioned. Weights (1) are used in all cases.

According to [10], the most appropriate variant is a conditional adjustment of a free network with a datum in Varna. The mean accidental error for a unit weight after adjustment μ is 1,21 $\text{mm}/\sqrt{\text{km}}$. As remarked in [6] nowhere the mean errors of the nodal bench marks are given even though these values are more representative concerning the accuracy of the levelling net than μ. In order to fix this omission, the results derived from a parametric adjustment of the network with a datum in Varna and weights (1) /variant 1 elsewhere below/, which is the same as the best variant of the adjustment according to [10], are shown in Table 1, Column 2. Variant 1 includes all 55 lines of the network, which form 23 loops. All heights which are used in the adjustment are corrected with a rod meter correction and turned into normal heights. The scheme of the network is given in Figure 1.

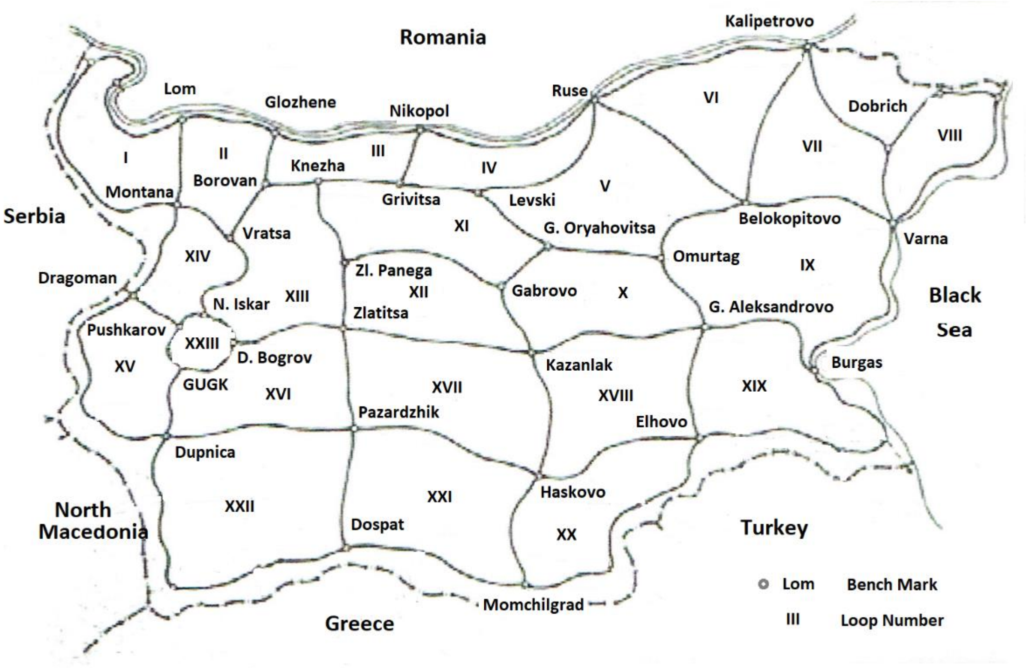

Figure 1. Scheme of the Third Precise Levelling Network of Bulgaria

2.2. Parametric adjustment with a condition

This is our variant 2, which differs from variant 1 only by an additional condition. In order to clarify utterly the procedure of this variant, a brief explanation is given below. Let v_{ij} are the corrections in the measured heights h_{ij} between bench marks I and j and their initial heights are H_i and H_j, respectively. Let x_i and x_j are the corrections of H_i and H_j. Then, our correction equations can be written as (3).

$$V_{ij} = \left(H_i + x_i\right) - \left(H_j + x_j\right) - h_{ij} = x_i - x_j + \left(H_i - H_j - h_{ij}\right) = x_i - x_j + f_{ij}\ . \tag{3}$$

Thus, in matrix form (3) can be presented by (4).

$$V = A.X + f\ . \tag{4}$$

In equation (4) A is an information matrix which contains the coefficients forward the unknowns x, X is a vector of the unknowns x and f is a vector of the free members in (3). Using the above symbols we can write our additional condition (5) and in matrix form (6).

$$\sum x_i = 0\,. \tag{5}$$

$$B.X = 0\,. \tag{6}$$

In (6) B is a vector which values are ones. Now our aim is to obtain the unknown x in accordance with conditions (6) and (7).

$$V^T PV \to \min . \tag{7}$$

In (7) P is a matrix of the weights. All members of P matrix are zeros except the members in the main diagonal, which are values of the weights of each levelling line calculated by (1). To obtain the corrections v_{ij} and x_i we use equation (8), where K is a vector of correlates. In our adjustment we have only one correlate.

$$Q = V^T PV + 2K^T BX \ . \tag{8}$$

After substitution into Lagrange's equation $dQ / dV = 0$ and some matrix manipulations we yield (9).

$$A^T PAX + B^T K + A^T Pf = 0 . \tag{9}$$

Let $N = A^T PA$ be a normal matrix and $A^T Pf = F$. Thus, we yield our extended normal system (10) or (11) and after its solution we will obtain the unknown corrections x.

$$\begin{vmatrix} N & B^T \\ B & 0 \end{vmatrix} . \begin{vmatrix} X \\ K \end{vmatrix} + \begin{vmatrix} F \\ 0 \end{vmatrix} = 0 . \tag{10}$$

$$N_e . X_e + F_e = 0 . \tag{11}$$

Using equation (11) one can yield the corrections v_{ij}. The mean error of the weight unit m can be calculated by (12), where $r = n - k = 55 - 32 = 23$.

$$m^2 = \frac{[PVV]}{r} . \tag{12}$$

The matrix $Q = N_e^{-1}$ is an extended covariance matrix. The first k = 32 values in the main diagonal of Q are the inverse values of the nodal bench mark weights. These values are treated as is described in (Kääriäinen 1953, p. 55 – 56) in order to receive the final values Q'_{ij} . Finally, the mean errors of the nodal benchmarks one can compute by (13).

$$m_i = m \sqrt{Q'_{ij}} . \tag{13}$$

3. Results

The mean errors of the nodal bench marks obtained in both variants of adjustment are given in Table 1 below. Table 1 also contains an approximate remoteness of each bench mark from the datum point in Varna.

Table 1. Mean Errors and Remoteness of the Bench Marks

Bench Mark	Mean Errors, mm		Remoteness, km
	Without Condition	With Condition	
Dobrich	7,21	7,22	52
Belokopitovo	9,41	7,57	111
Kalipetrovo	10,32	9,32	149
Burgas	11,53	9,79	152
Omurtag	11,17	7,96	171
Ruse	11,73	8,71	223
G. Aleksandrovo	11,73	8,05	229
G. Oryahovitsa	12,65	7,90	256
Elhovo	13,33	9,40	294
Gabrovo	13,37	7,79	321
Levski	12,97	8,01	348
Kazanlak	13,26	7,47	363
Nikopol	13,81	8,49	367
Grivitsa	13,41	7,56	394
Haskovo	14,20	8,70	416
Knezha	14,00	6,76	460
Zl. Panega	14,22	7,27	462
Zlatitsa	14,23	6,69	493
Borovan	14,49	6,86	497
Glozhene	14,94	7,92	498
Vraca	15,02	7,29	531
Pazardzhik	15,01	8,22	550
Momchilgrad	15,56	10,69	552
Lom	15,92	9,10	558
D. Bogrov	15,12	6,98	566
Montana	15,49	8,02	575
N. Iskar	15,17	6,94	584
GUGK	15,27	7,13	597
Pushkarov	15,22	6,96	598
Dragoman	15,78	8,14	636
Dupnitsa	15,79	8,51	658
Dospat	16,47	10,72	750

Table 2 contains the results from Paired Two Sample for Means t-Test, which was applied in order to compare the results yielded in both adjustments. The null hypothesis is that the means of mean errors of identical bench marks produced in both variants of adjustment are equal – H_0: $\mu_1 = \mu_2$.

Table 2. t-Test: Paired Two Sample for Means

	Without Condition	With Condition
Mean	13,68 mm	8,07 mm
Median	14,21 mm	7,94 mm
Variance	4,40 mm^2	1,13 mm^2
Observations	32	32
Pearson Correlation	0,07	
Hypothesized Mean Difference	0,00	
df	31	
t Stat	13,91	
P(T<=t) one-tail	$3{,}6.10^{-15}$	
t Critical one-tail	1,70	
P(T<=t) two-tail	$7{,}2.10^{-15}$	
t Critical two-tail	2,04	

Table 3 contains the coefficients of correlation between the mean errors produced in both variants and their remoteness from the datum point.

Table 3. Coefficients of Correlation between the RMSE of the Bench Marks and Remoteness from the Datum Point

	Without Condition	With Condition	Remoteness
Without Condition	1,00		
With Condition	0,07	1,00	
Remoteness	0,95	-0,02	1,00

Table 4 contains the differences between the adjusted heights of each nodal bench mark derived in both adjustments and remoteness of the bench mark from the datum point. All differences are calculated by equation (14).

$$D_i = H_i^{\text{Variant 1}} - H_i^{\text{Variant 2}}. \tag{14}$$

Table 4. Differences in the heights of the bench marks obtained by both variants

Bench Mark	Height Difference, mm	Remoteness, km
Dobrich	-1,33	52
Belokopitovo	-4,91	111
Kalipetrovo	-4,15	149
Burgas	-5,48	152
Omurtag	-6,75	171
Ruse	-6,81	223
G. Aleksandrovo	-7,31	229
G. Oryahovitsa	-8,43	256
Elhovo	-8,14	294
Gabrovo	-9,23	321
Levski	-8,69	348
Kazanlak	-9,29	363
Nikopol	-9,28	367
Grivitsa	-9,40	394
Haskovo	-9,56	416
Knezha	-10,36	460
Zl. Panega	-10,33	462
Zlatitsa	-10,60	493
Borovan	-10,77	497
Glozhene	-10,72	498
Vraca	-11,09	531
Pazardzhik	-10,65	550
Momchilgrad	-9,73	552
Lom	-11,10	558
D. Bogrov	-11,32	566
Montana	-11,22	575
N. Iskar	-11,38	584
GUGK	-11,40	597
Pushkarov	-11,42	598
Dragoman	-11,44	636
Dupnitsa	-11,27	658
Dospat	-10,71	750

4. Discussion

The main aim of this manuscript is a comparison of the efficiency of two approaches of adjustment of precise levelling nets. The first one is a parametric least squares adjustment of a free network. This variant of adjustment is identical with a conditional least squares adjustment of a free network, which variant is assessed as the best one in terms of the Third Levelling of Bulgaria [10]. The second variant is a parametric least squares adjustment with a condition which is given in [11, p. 55 – 56]. The mean accidental error for a unit weight derived from both adjustment is equal to 1,21 $\mathrm{mm}/\sqrt{\mathrm{km}}$ and 1,23 $\mathrm{mm}/\sqrt{\mathrm{km}}$ for variants 1 and 2, respectively. Nevertheless, the second variant should be preferred. The main arguments which support this opinion are:

- The mean errors of the nodal bench marks in the network yielded by variant 2 are significantly less than these of variant 1. Looking at Table 1 one can see that 30 out of 32 bench marks have mean errors greater than 10 mm in the case of variant 1. This fact was remarked in [6]. In contrast, only 2 out of 32 mean errors are greater than 10 mm in the case of variant 2. According to Table 2, one can claim that the accuracy obtained in variant 2 is higher than those of variant 1 on confidence level much greater than 99,99 %. The median of the mean errors is 7,94 mm against 14,21 mm to the advantage of variant 2.

- Looking carefully at Table 1 one can see that the mean errors of the nodal bench marks produced by variant 1 increase with distancing from the datum point in Varna. Using the data from Table 1 the reader is able to construct a regression model in form ME(variant 1) = 2,717 + 0,7506 $\sqrt{L}$ – 0,0094 L, which gives the relation between the mean errors of the nodal bench marks and their remoteness from the datum. Using the results from variant 2 the regression model can be given as ME(variant 2) = 8,213 – 0,0074 $\sqrt{L}$. Thus, if a bench mark is a 1000 km far away from a datum its mean error will be approximately the same as the median value of the mean errors derived from the second adjustment approach. Representative picture of the relationship between mean error of each benchmark and its remoteness from the datum is illustrated by the correlation coefficients given in Table 3. According to Table 3, there is an almost functional relationship in the case of variant 1 and there is not any correlation in the case of variant 2.

Another important question is how big are the differences in the adjusted heights of the bench marks yielded by both methods? According to Table 4, these differences can be over 10 mm, which is close to the mean errors of the bench marks. Looking at Table 4, one can see that the adjusted heights from variant 1 are less than these obtained in variant 2. Also, a rise in the amount of the absolute values of the differences can be distinguished with distancing from the datum. Supposing that the results obtained by the second adjustment approach are significantly more accurate than these of the first one, which is actually illustrated by the data in Table 1, one can conclude that there is a systematic decrease in the direction of west in the official accepted adjustment of the Third Levelling in Bulgaria.

5. Conclusion

The precise levelling is an expensive and time-consuming method. On the other hand, this method has been applied in important scientific [12, 13] and engineering tasks [14, 15].

Therefore, we have to try to develop this method not only by applying modern digital levels and bar-coded rods but also by searching for better weight models and adjustment procedures than traditionally used in our country. Talking about the improvement of the mathematical treatment of levelling data, it is high time for the available information [7] to be published and accepted by independent researchers [2]. An appropriate format should be similar to the one used in [5].

REFERENCES

1. Instruktsia № RD-02-20-1 ot 15 yanuari 2021 g. za sazdavane i poddarzhane na darzhavnata nivelachna mrezha. V sila ot 05.02.2021 g., Izdadena ot ministara na regionalnoto razvitie i blagoustroystvoto, Obn. DV. br. 10 ot 5 Fevruari 2021 g.

2. *Gospodinov, S., Belyashki, T., Peneva, E.* Analiz na darzhavnata nivelachna mrezha na Republika Balgaria. Doklad na rabotna grupa po zadaca 3. Syzdavane na programa i usavarshenstvane na darzhavnite geodezicheski mrezhi (3.2. Darzhavna nivelachna mrezha) kym Saveta po geodezia, kartografia i kadastar, 2014.

3. *Gospodinov, S., Peneva, E., Penev, P., Lambeva, T., Tsanovski, Y., Dzhorova, S., Marinov, G., Radev, I.* Savremenni aspekti na geometricnata nivelacia. // Godishnik na Universiteta po arhitektura, stroitelstvo i geodezia – Sofia, 2016, Vol. 49, Issue 4, 9 – 23, ISSN 1310-814X.

4 Instruktsia za nivelatsia I i II klas. Glavno upravlenie po geodezia, kartografia i kadastar, Sofia, 1980.

5. *Kääriäinen, E.* The Second Levelling of Finland in 1935 – 1955. Publications of the Finnish Geodetic Institute No. 61, Helsinki, 1966.

6. *Naydenov, Hr.* Niakoy syobrazhenia po nivelatsiata I klas na NR Bulgaria, Geodezia, kartografia i kadastyr, 1988, 6: 13 – 20.

7. Geokartfond – nivelachen arhiv.

8. *Gekov, D., Tzvetkov, V.* The problem about choice of weights in the adjustment of precise leveling networks. Proceedings of the International Symposium on Modern technologies, education and professional practice in geodesy and related fields, Sofia, 2005, 290 – 296 (in Bulgarian).

9. *Tzvetkov, V.* Izravnenie na chast ot pyrvoklasnata nivelachna mrezha na R Bulgaria /treto izmervane/ s razlichni tezhestni modeli. // Godishnik na Universiteta po arhitektura, stroitelstvo i geodezia – Sofia, 2005, Vol. XLI, Issue 3, 19 – 25 (in Bulgarian).

10. *Burilkov, T., Vuchkova, R.* Izravnenie na novata nivelachna mrezha I klas na N. R. Bulgaria, Geodezia, kartografia i zemeustroystvo, 1986, 4: 7 – 9.

11. *Kääriäinen, E.* On the recent uplift of the Earth's crust in Finland. Publications of the Finnish Geodetic Institute No. 42, Helsinki, 1953.

12. *Beliyashky, T.* Analiz na razlikite vyv visochinite na nivelachnite reperi I klas mezhdu epohi 1982 g. i 1958 g., Geodezia, kartografia i zemeustrojstvo, 1989, 4: 21 – 22.

13. *Kanev, D., Mladenovski, M.* Syvremenni vertikalni dvizhenia na zemnata kora v Bulgaria, Izvestia na bulgarskoto geographsko druzhestvo, 1969, Vol. IX, 17 – 26.

14. *Angelov, A.* Geodezicheski metodi za izsledvane na deformacionni procesi pri visoki sgradi i inzhenerni saorazhenia. UACEG, 2017, https://uasg.bg/filebank/att_13033.pdf, poseten na 16.07.2022.

15. *Angelov, A., Penev, P.* Preliminary assessment of the accuracy of the tunnel engineering-geodetic network. // Godishnik na UASG, 2017, 50 (2): 251 – 259.

РЕДУЦИРАНЕ НА РАЗПРОСТРАНЕНИЕТО НА ГРЕШКИТЕ ПРИ ИЗРАВНЕНИЕТО НА ТРЕТАТА НИВЕЛАЦИЯ НА Р БЪЛГАРИЯ

А. Ангелов[1], В. Цветков[2]

Ключови думи: *прецизна нивелация, изравнение, предаване на грешки*

РЕЗЮМЕ

Цел на настоящата публикация е да сравни резултатите, получени от два метода на изравнение на Третата нивелация на България. Първият метод е параметрично изравнение на мрежата като свободна. Вторият метод е също параметрично изравнение на мрежата като свободна, но с поставяне на условието сумата от корекциите на приблизителните височини на възловите репери при изравнението да е равна на нула. За изходен репер и при двата варианта е приет възловият репер във Варна. Сравнението на резултатите от двете изравнения показва, че средните квадратни грешки на възловите репери, получени при втория вариант, са два пъти по-малки в сравнение със средните квадратни грешки, получени при изравнението без условие. При метод 1 е установено увеличаване на средните квадратни грешки на изравнените височини на възловите репери с отдалечаване от изходния репер. Такова поведение не се наблюдава при втория метод на изравнение на мрежата. При сравняване на абсолютните разлики в изравнените височини на възловите репери, получени от двата метода, е констатирано, че половината от тези разлики са по-големи от 10 mm, което е стойност, по-голяма от стойностите на средните квадратни грешки във височините на реперите, получени при метод 2.

[1] Антонио Ангелов, доц. д-р инж., кат. „Приложна геодезия“, УАСГ, бул. „Христо Смирненски“ № 1, София 1046, e-mail: angelov_fgs@uacg.bg

[2] Васил Цветков, ас. д-р инж., кат. „Технология и мениджмънт в строителството“, ВСУ „Любен Каравелов“, ул. „Суходолска“ № 175, София 1373, e-mail: tzvetkov_vasil@abv.bg