

APPROXIMATE DESIGN OF SLENDER BI-AXIALLY LOADED RC COLUMNS

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ABSTRACT

Due to high complexity of general procedures, practical design of slender bi-axially loaded RC columns uses simplified approaches, usually related to numerous concerns and inconsistencies. Since such columns are unavoidable in building structures, more accurate design treatment is still of current interest.

Following the results of voluminous numerical experiment, based on the most general design approach, the most influential parameters affecting the behaviour of slender columns are recognized and their effects are quantified. Derived from such investigations, a proposal of approximate ultimate limit design of bi-axially loaded slender RC columns is formulated. The simplified procedure is straight-forward, does not employ iterative calculations and, therefore, is intended and is convenient to be used in everyday engineering application.

1. Introduction

Modelling the behaviour of compressed slender reinforced concrete columns in ultimate limit state is numerically very demanding problem and dependent on large number of parameters. Non-linear constitutive relations for concrete and reinforcement make the

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analysis materially non-linear, while element/structural deformation may significantly influence the level of stresses and may not be neglected. Therefore, such elements should be analysed in the way that their geometrically non-linear behaviour and second order effects are accounted. When columns are subjected to biaxial bending, which is the most common case in practice, the problem becomes much more complex and demanding.

RC elements' design procedures are, in practice, still based on first order effects, where the equilibrium corresponds to non-deformed element and where effects of material non-linearity are introduced at the dimensioning stage, locally, and do not affect stress distribution. In practice, such approach is highly justified for members not sensitive to deformational effects. However, when slender columns are concerned, design procedure has to be upgraded in the way to account, not only second order effects, but other deformational phenomena, such as imperfections or creep deformation, as well.

Multi-storey buildings are usually designed as being horizontally braced. Beside serviceability requirements, such practice is justified in order to keep the second order effects low enough. Thus, building in most cases may be classified as non-sway structure, with possibility to completely neglect horizontal displacement of its joints. Criteria classifying such approximation, although rough, are necessary regarding much more complex alternatives, inconvenient for practical use. Now, moments at the column's ends are not dependent on second order structural behaviour which allows separate analysis of isolated columns: only local analysis of second order effects.

Furthermore, for bi-axially bended columns, provisions of technical codes mainly stay unsaid and suggested design procedures are characterised with inconsistency and absence of clear physical background. Since the application of general procedures, which consistently use basic design assumptions, is still highly limited with development of computers, approximate procedures come as an only alternative and their formulation – a necessity.

A part of investigations performed, among other, for the purpose of formulating alternative design procedure [1], are presented in the paper. Analysis is limited only to non-sway columns of rectangular cross-section having symmetric reinforcement layout. The columns are pinned at their ends where they are subjected to axial load of constant eccentricity. Eurocode design assumptions and provisions are used. Since the analysed problem simultaneously includes two simpler ones – biaxial bending of cross section and buckling of slender columns, the paper structure follows that scheme. As a result of performed analysis, approximate procedures, a proposal that may be applied during practical design of such elements, are given.

2. Bi-axial bending at cross-sectional level

When acting moment angle does not coincide with any of main directions, cross-section is bi-axially bended and direction of neutral axis (bending angle) do not coincide main directions nor acting direction – it is inclined towards weak main axis (Figure 1). Resulting stress/strain state in RC cross-section, beside geometric, is dependent on mechanical properties of steel and concrete and on layout and amount of reinforcement. Ultimate resistance of section may be calculated in the form of interaction surface in M_x - M_y - N (N -axis is vertically oriented) coordinate system (Figure 2).

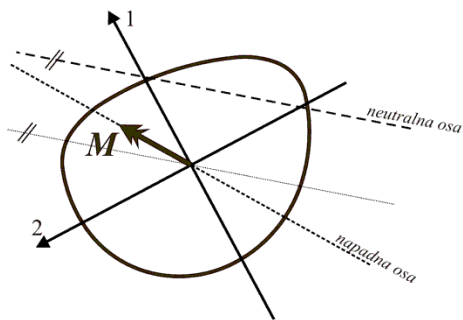


Figure 1. Acting and bending axis of biaxially bended section

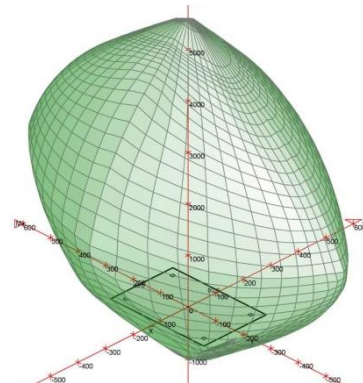


Figure 2. Interaction surface for symmetrically reinforced rectangular section

Such presentation, although not giving information about bending angle or strains, from the practical point of view, is of greatest importance – external effects, presented by one point in 3D coordinate system, could be resisted by the section if the point is located inside or on a surface. Furthermore, the procedure that leads towards interaction surface determination is inevitable followed with determination of not shown data. Numerically, one stress-strain state of cross-section, defining effects-point, corresponds to considered inclination and location (along the height) of neutral axis. By varying location and inclination, it is possible to determine any point on interaction surface. On the contrary, dimensioning problem is more complex and assumes the determination of inclination and location of bending axis of the section of known only layout (not amount) of reinforcement. In such case, it is performed throughout iterative procedure in search for moments and force equilibrium (Figure 3).

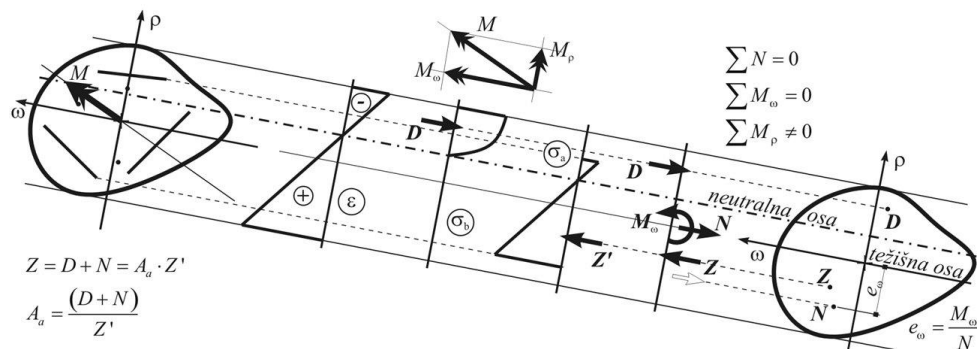


Figure 3. Equilibrium of external and internal forces for one bending angle

3. Uni-axially bended slender columns

Slender columns are those compressed, subjected to bending and having ultimate resistance significantly influenced by deformations. Equilibrium state and resistance of such members should be analyzed having deformed model in front. Beside effects of geometrical

nonlinearity, in such occasions, all the effects that significantly influence deformation have to be accounted: non-linear material properties, geometrical imperfections, stiffness reduction due to cracking, rheological effects...

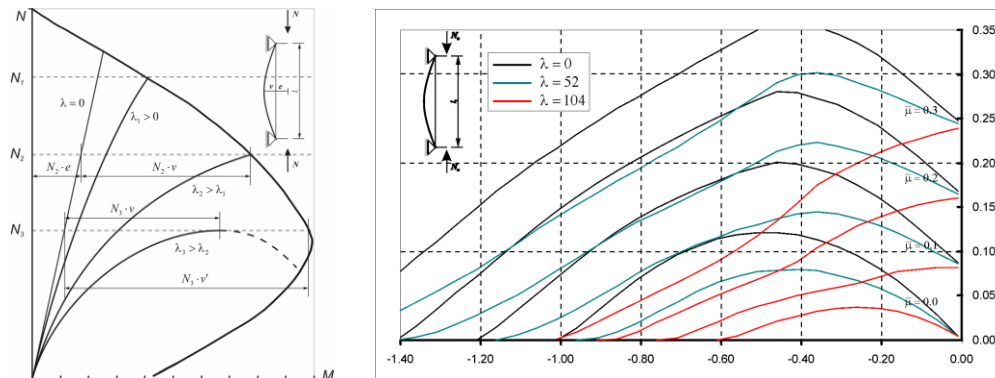


Figure 4. Interaction diagram for columns of different slenderness

The nature of column's sensitivity to local deformation effects is easy to realize using interaction diagram corresponding to cross-section of the column. Standard interaction curve gives the ultimate resistance of perfectly short column, while, for "real" columns, a fall of ultimate resistance increases with slenderness ratio (relative effective length of the column) of column. It is presented in the form of interaction diagram for slender columns in Figure 4. Interaction curves are related to first order effect column may be exposed to.

In Eurocode [2], methods of analysis of slender columns are classified in three groups: general method, which is based on non-linear analysis of second order and which may be used as referent for verification of approximate methods; methods based on nominal stiffness of elements and; methods based on prediction of nominal curvature of critical section and/or its distribution. Within *general method*, the general simulative approach is assumed, which simultaneously accounts effects of material and geometrical non-linearity, as well as geometrical imperfections and other effects that notably influence column's deformation. Effects of material non-linearity are taken into account through constitutive relations for concrete and steel while calculation procedure is iterative. Approximate procedures based on *nominal stiffness* are most suitable for application within structural analysis and they tend to predict moments of second order by multiplication of first order ones with factor depending on estimated stiffness of members. Approximate procedures based on *nominal curvature* are convenient for application for isolated members. Nominal second order moments are determined after assumption of curvature distribution its value in critical section. Model-column method is the suggested procedure of this type.

4. Bi-axial bending of slender columns

In codes and standards, design of bi-axially loaded slender columns is covered only roughly, assuming the separation of an analysis into two main direction analyses. General method, obviously, may always be applied. However, its application is numerically very demanding so the approximate procedures come as only alternative in practical designs. In Eurocode, separate directional analysis is suggested as a first step where imperfections

should be applied only in less favourable direction. In second step, only check of resistance when cross-section is exposed to moments increased due to directional deformational effects is assumed. Such approach does not include effects arising from the fact that two directions are “acting” simultaneously and may hardly be justified from the engineering point of view.

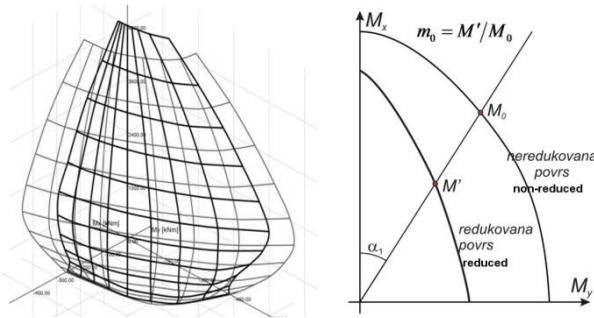


Figure 5. Non-reduced and reduced surface

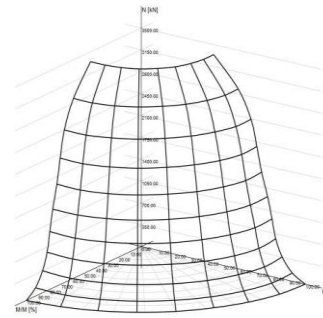


Figure 6. Relative bending moment and relative interaction surface

In case of bi-axially bended column, instead of using interaction curves (as in Figure 4), ultimate limit resistance of column may be presented in the form of interaction surface for slender column – *reduced interaction surface* (*non-reduced surface* is related to perfectly short column). It defines ultimate limit effects of first order that, after realization of second order and deformational effects, would lead column in ultimate limit state by loss of resistance or, rarely, by loss of stability. In Figure 5, two surfaces, reduced and non-reduced one, are plotted simultaneously. Spectra of reduced surfaces, for column's varying in length, correspond to one non-reduced surface.

It is convenient to show reduced surface in relative way – ratios of reduced to non-reduced bending moments, which lay on the same acting direction and for a given axial force (Figure 6, left). Obtained surface is named *relative reduced surface* (Figure 6, right). Having such surface known, ultimate limit resistance of bi-axially bended column may be determined on the basis of known non-reduced surface.

5. Numerical experiment

In the form of numerical experiment, for columns differing in sectional dimensions and ratios, concrete quality, reinforcement layout and its amount, reduced interaction surfaces are determined through use of general approach, which simultaneously accounts effects of material and geometrical non-linearity. In iterative procedure, for given geometrical and material data, resulting stress-strain state for series of cross sections along column's length is determined, and then, after double integration of curvature distribution, deformation state is determined. In next iteration changes in moment distribution due to deflections are accounted. Iterative procedure ends when two consequent iterations “coincide” in deflections (very sharp tolerances were used) or when the column's failure is recognized. Eurocode constitutive relations were used, while partial coefficients are taken to correspond to permanent or transient situations.

Table 1. Analyzed columns

Length or slenderness	Concrete	Cross section	Reinforcement layout	Steel ratio
[m] or [-]	[-]	[cm/cm]	[-]	[%]
3, 5, 7, 9, 11 m	C30	40/40, 40/60, 40/80, 40/100	dot, uniform	0.8, 1.4, 2.0%
20, 40, 60, 80, 100 m	C30	25/50, 30/45, 30/60, 40/50, 40/120, 60/90	dot, uniform	1.0, 2.0, 3.0%
2, 4, 6, 8, 10, 12 m	C30, C40, C50	40/40 and 40/80	uniform	1.0, 2.0, 3.0%

In total, 288 columns are analyzed (Table 1). Reinforcement quality is constant (S500). Two analyzed reinforcement layouts correspond to reinforcement grouped in section corners and steel equally distributed along section perimeter (in both cases, lines and dots of reinforcement are placed at 5 cm from the edges). As most important result in numerical experiment, reduced interaction surfaces, as well as relative ones, are obtained.

6. Critical force and resistance distribution approximation

Critical force, in performed investigations, is the level of axial force that leaves a column with no moment resistance (regardless of type of failure). Its value is a function of geometrical and material properties of column and section and its change with slenderness is shown in a qualitative way in Figure 7.

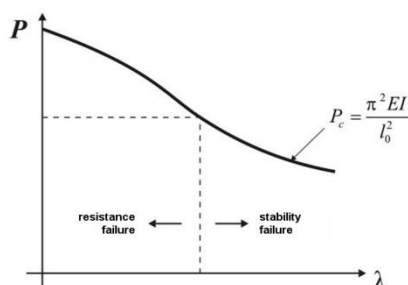


Figure 7. Change of critical force of uni-axially bended column with slenderness – qualitative

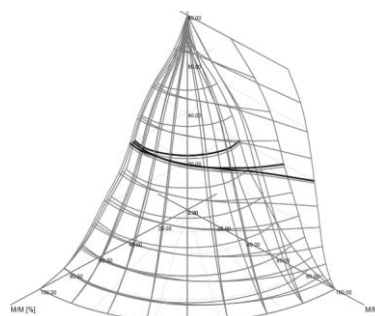


Figure 8. Relative surfaces for different depth-width ratios of cross section

When bi-axially bended column is concerned, in general, there is one, different, value of critical force for each acting direction. It has maximum value for uni-axial bending along strongest axis, while minimal – for bending along weakest axis. Both, strongest and weakest, axes are main ones. The difference between these two values increases with sectional depth-to-width ratio increase. In Figure 8, relative surfaces for three depth-to-width ratios (1.0, 1.5 and 2.5) are shown. Region of large values of critical force is limited only to narrow interval near the zero acting angle (bending along strongest axis). Having in mind inevitable imperfections of loading and geometry, it is justified and conservative, for practical purposes, to neglect existence of such region. In the rest of interval, towards acting angle of $\pi/2$, critical force is characterized with relatively slow change, which implies the possibility of its adopting as being constant for column, regardless of acting directions. Since

approximate procedures should be characterized as conservative, the lower value (weakest direction) of critical force may be taken, which is used in presented analysis.

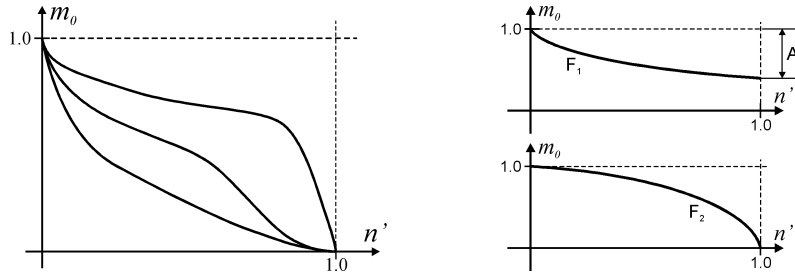


Figure 9. Normalized relative curves $m_0(n')$; Shape of component functions

Numerically, critical force value is not easy to determine. At the curve of relative uni-axial resistance, theoretically, all curves lose moment resistance ($m_0 = 0$) only for $n = 1$. That's why, value of critical force, when column's moment resistance is lost, mostly depends on the way of its determination. High calculation sensitivity causes large results dispersion. Knowing the critical force enables determination of curves in m_0-n' coordinate system, providing that all relative curves lie in the same field of relative values – between 0 and 1, along both axes (Figure 9, left). Parameter n' is ratio between acting axial and critical force. Procedure of determination of critical force is performed by use of numerical approximation of experimentally obtained curves of relative resistance. For such purpose, function (F) has been approximated as a product of two functions of shown shape (Figure 9, right):

$$F = (1 - A \cdot n'^B) \cdot \frac{1 - \exp(-C \cdot (1 - n'))}{1 - \exp(-C)}, \quad n' = \frac{n}{n_c} \quad (1)$$

Parameters A , B , C and n' are numerically determined by keeping the average square deviation at minimum and favoring higher precision at the region of critical force. Analysis of results obtained show small influence of sectional depth to width ratio and reinforcement layout and amount. Therefore, its dependence only on slenderness ratio is adopted. Curves of critical force change are numerically approximated, in the field of whole experimental specimen, using the following function:

$$n_c = (2/\pi) \arctan(C/l^2), \quad l = L/d, \quad (2)$$

where n_c is relative critical force, L is column's length, d is height of cross section (in this case shorter one), l is slenderness parameter, while C is constant. As a result of numerical approximation of numerically obtained curves, a constant value of $C = 950$ is obtained.

With known critical force (constant for column), the possibility of approximate determination of uni-axial resistance of the column is researched, as an alternative to general or model-column procedure. It is performed by use of numerical approximation of relative resistance curve (Figure 9). For such purpose function (1) is not convenient anymore, having whole n' domain in mind: significant deviations are registered, function is set as being four-parametric and their physical differentiation could not be recognized. After numerous attempts with different analytical expressions, being most acceptable when accuracy and convenience of application is concerned, the following, rather rough expression to approximate relative resistance came out (Figure 10):

$$p_0 = 1 - m_0 = A \cdot \left(\tan(0.5 \cdot \pi \cdot (1 - n_0)) \right)^{-B}. \quad (3)$$

Value p_0 is named the fall of resistance. Through voluminous numerical analysis, based on minimization of average square deviation, it is determined:

$$A = P \cdot \ln \left[\left(\frac{l}{Q} \right)^2 + 1 \right], \quad B = 0.6 \cdot (1 + \bar{d}) \cdot f^{0.35}, \quad f = \mu \cdot f_{yd} / f_{cd} \quad (4)$$

$$P = 1.44 \cdot (1 - \bar{d}) \cdot f^{0.5}, \quad Q = 27.14 \cdot f^{0.35}, \quad \bar{d} = 0.0375 \cdot (d/b - 1), \quad f = \mu \cdot f_{yd} / f_{cd}. \quad (5)$$

Though obtained expressions are not of compact form and the approximation, itself, is set in rather rough way, deviation of approximate values form the experimentally determined ones are not large (Figure 11), especially in the regions of “real” loads.

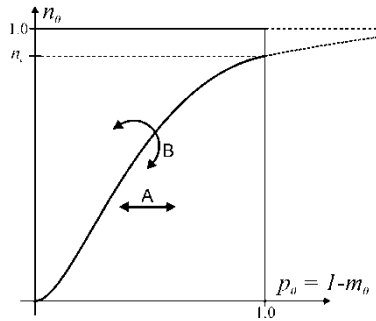


Figure 10. Adopted shape of approximation of uni-axial resistance

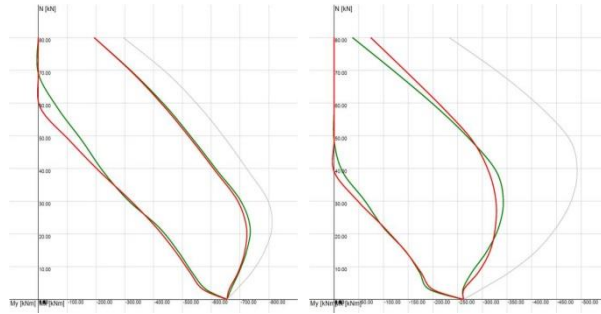


Figure 11. Reduced moments determined by application of approximation (3)

Finally, possibility of approximate defining the moment resistance change with change of acting angle is investigated. Relative reduced resistances for two main directions are marked with m_0' , for $\alpha' = 0$, and m_0'' , for $\alpha' = \pi/2$. Corresponding falls of resistance are marked with p_0' and p_0'' . Normalized relative resistance fall, for one value of axial force, may be defined in the following way (Figure 12):

$$\bar{p}(\alpha') = \left(p_0(\alpha') - p_0' \right) / \left(p_0'' - p_0' \right). \quad (6)$$

Through numerical analysis, possibility of its approximation by power function is approved (when critical force value is known) (7), and change of parameter A with axial force is approximated in the following way, by linear function (Figure 13):

$$\bar{p} = (2 \cdot \alpha' / \pi)^A, \quad A = A_0 \cdot (1 - n') \quad (7)$$

Parameter A_0 is constant for a column and given expression for its determination comes as a result of statistically based numerical analysis, bringing it into relation to parameters that mostly influence it: column's slenderness and sectional depth-to-width ratio. By use of such approximation, high accuracy level is achieved (Figure 14), when reduced interaction surfaces are concerned. More significant deviations are registered only in regions near critical force and come as a consequence of previously adopted approximation – constant critical force for one column.

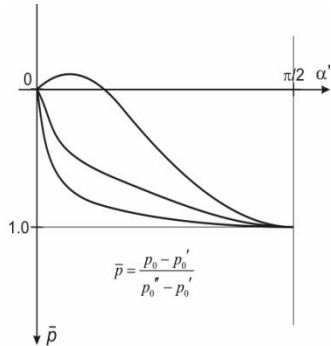


Figure 12. Relative decrease of resistance – normalization

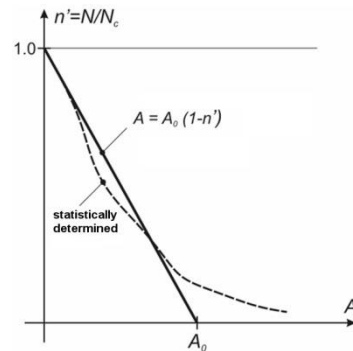


Figure 13. Linear approximation of relation $A(n')$

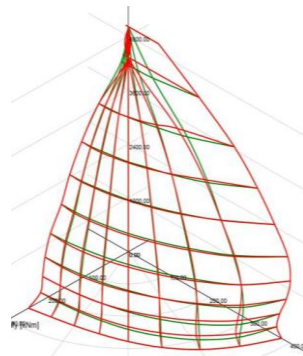


Figure 14. Reduced approximated surface

7. Formulation and accuracy of approximate procedure

Summing the presented approximations leads toward proposal of approximate procedure which consists from the following steps:

- Approximate determination of critical force as a column's property. This approximation is introduced by expression (2).
- Approximate determination of uni-axial resistance distributions along n -axis for two main directions. This approximation is presented by expressions (3) to (5).
- Approximate determination of angular (acting angle) change of moment resistance as it is presented by expressions (6) and (7).

Result of application of such approximation may be reduced surface or just one point of a surface, both in relative values. Extracting the absolute values is possible on the basis of known non-reduced surface/value.

A representative set of 16 columns (Table 2) is used in testing the accuracy of proposed procedure. Chosen columns vary in length, height of the cross section (width is

constant, 40 cm), reinforcement ratio and effective creep coefficient. For all samples, concrete class is C30, while steel class is S500. Columns are numbered as it is shown in the table.

Results obtained approximately are compared to ones from numerical experiment. Comparison is performed in the way that maximal relative deviations of reduced interaction surfaces are registered. Absolute deviation is normalized by corresponding non-reduced value (Figure 15, left):

$$\Delta = \frac{|M' - M|}{M_0} = |m'_0 - m_0|. \quad (8)$$

Table 2. Representative set of columns

No.	L [m]	d [cm]	μ [%]	φ_{eff} [-]	No.	L [m]	d [cm]	μ [%]	φ_{eff} [-]
Col. 1	5.0	60.0	0.8	0.0	Col. 9	5.0	60.0	0.8	2.0
Col. 2	9.0	60.0	0.8	0.0	Col. 10	9.0	60.0	0.8	2.0
Col. 3	5.0	60.0	2.0	0.0	Col. 11	5.0	60.0	2.0	2.0
Col. 4	9.0	60.0	2.0	0.0	Col. 12	9.0	60.0	2.0	2.0
Col. 5	5.0	100.0	0.8	0.0	Col. 13	5.0	100.0	0.8	2.0
Col. 6	9.0	100.0	0.8	0.0	Col. 14	9.0	100.0	0.8	2.0
Col. 7	5.0	100.0	2.0	0.0	Col. 15	5.0	100.0	2.0	2.0
Col. 8	9.0	100.0	2.0	0.0	Col. 16	9.0	100.0	2.0	2.0

In practice, not all regions along the axial force axis are of same importance. That's why maximal deviations are investigated in four different intervals simultaneously along n' -axis (axial force relative to critical force), as shown in Figure 15, right):

- Interval 1: $n' = 0.00 \div 1.00$; whole domain,
- Interval 2: $n' = 0.00 \div 0.80$;
- Interval 3: $n' = 0.00 \div 0.60$; and
- Interval 4: $n' = 0.15 \div 0.60$; the most common case.

The narrower the interval – the smaller (or, at least, not greater) the maximal deviation is.

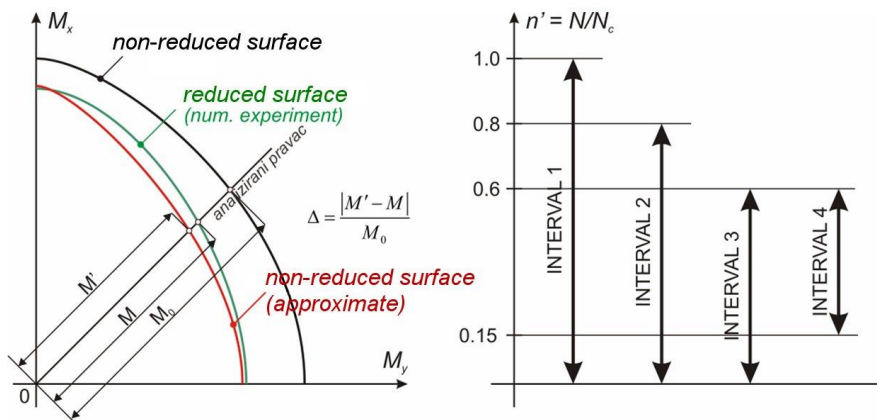


Figure 15. Definition of relative deviation and intervals analyzed

Results of accuracy analysis, for columns from representative set, are shown in Figure 16, for all four intervals analyzed. Only maximum relative deviations are registered and plotted along the analyzed interval, regardless of acting direction it occurs.

The obtained results show that, excluding very high axial forces, which are close to the critical one (Interval 1), absolute maximums of measured relative deviations even in Interval 2 do not exceed 10%. For the narrower intervals, absolute maximums are reduced further, under 7%, and mostly correspond to the narrowest one.

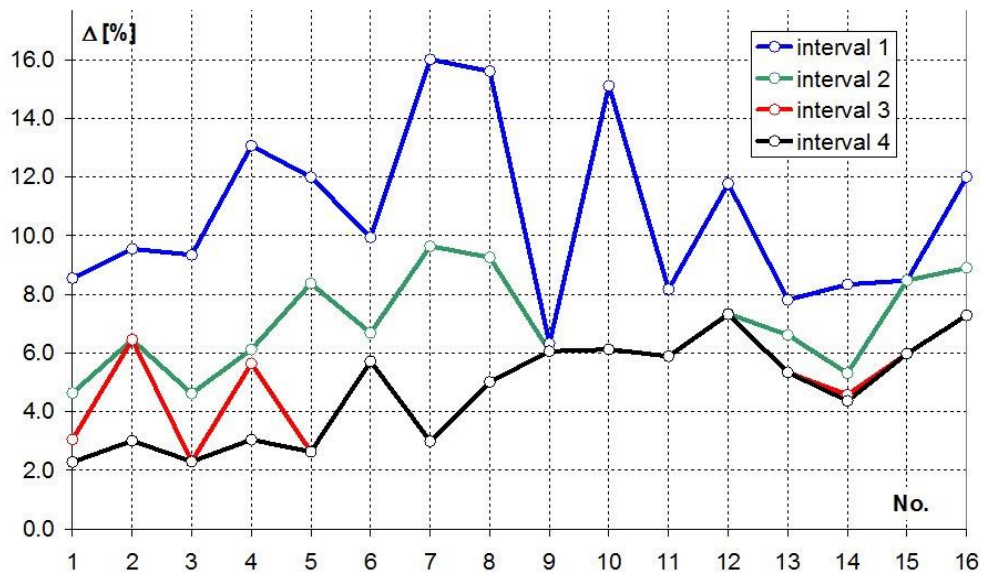


Figure 16. Accuracy analysis – maximal deviations (relative)

8. Final remarks

Numerical investigations which are performed in order to give clear presentation of behavior of slender bi-axially bended columns when in ultimate limit state are shortly presented in the paper. They aimed to identify the most important parameters and to result with proposal of approximate procedures for treating such members in practice. Starting from general procedure, numerous possibilities of simplification of its certain steps with less demanding procedures are considered. Approximate design proposal presented in the paper is the result of statistical approximations of interaction surfaces obtained using general design procedure. Approximations are introduced at three levels. First, critical force is set as a column property and its determination is set in simplified direct manner. Then, distribution of moment ultimate resistance along the axial force axis, in two main directions, follows direct numerical simplification. Finally, angular change of moment resistance is set in direct numerical expression using power function.

Although proposed procedure only roughly simulate ultimate resistance of slender column, accuracy analysis show practically acceptable deviations. However, parameters used within individual approximations are not completely physically differentiated, while only moment resistance is concerned (state of deformations remains unknown). Furthermore, one should be aware of initially limited domain of columns considered. On the other hand, all limitations introduced are commonly used when similar investigations are performed.

Acknowledgements

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